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The Limited Scaling Range of Empirical Fractals

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A notion which has been advanced intensively in the past two decades, is that fractal geometry describes well the irregular face of Nature. We were prompted by Marder's recent article in Science [1], to comment here on the applicability of this wide-spread notion. Marder summarizes a simulation study of fractured silicon nitride by Kalia *et. al* [2] which successfully mimics experimental data, and emphasizes the role of fractal geometry in describing complex-geometry physical structures, in general. Specifically, the results of Kalia's *et. al* were interpreted as "showing that this mechanism... leads to fractal fracture surfaces". However, upon examining Kalia's results (Fig. 4 in Ref. [2]) one finds that Marder's statement is based on four exponents, all of which hold over less than one order of magnitude. We recall that a fractal object, in the purely mathematical sense, requires infinitely many orders of magnitude of the power law scaling, and that a consequent interpretation of experimental results as indicating fractality requires, "many" orders of magnitude. We also recall that, for instance, in the celebrated fractal Koch flake, one order of magnitude means about two iterations in the construction and that such two-iterations Koch curve is not a fractal object. It is our feeling that Marder, like many others in the scientific community, may have been swayed by the wide spread image and belief that many-orders fractality abounds in experimental documentation.

In a recent detailed statistical data analysis we have shown that this is not the case, at

least in the original sense of the concept [3]: We found that reported experimental fractality in a wide range of physical systems is typically based on a scaling range which spans over only 0.5 - 2.0 decades. The survey was based on all experimental papers reporting fractal analysis of data which appeared over a period of seven years in all Physical Review journals (Phys. Rev. A to E and Phys. Rev. Lett., 1990 - 1996). In these papers an empirical fractal dimension, D, was calculated from various relations between a property, P, and the resolution, r, of the general form

$$P = k\, r^{f(D)} \tag{1}$$

where k is the prefactor for the power law and the exponent is a simple function of D. In most cases, fitting the data to Eq.(1) was done through its linear log-log presentation. Typically, the range of the linear behavior terminated on both sides either because further data is not accessible or due to crossover bends. A histogram of the number of orders of magnitude used to declare fractality, covering all of the 96 relevant reports, was prepared and is reproduced in Fig. 1. A clear picture emerges from it: the scaling range of experimentally declared fractality is extremely limited, centered around 1.3 orders of magnitude, spanning mainly, as mentioned above, between 0.5 and 2.0 [4]. This stands in stark contradistinction to the public image of the status of experimental fractals.

It seems that the most acute questions to be asked in view of this data are: Is the limited range inherent?; are these limited range power-law objects, fractal?; and, in fact, is nature describable in terms of fractality? For reasons detailed next, we are inclined to propose that the question of fractality is secondary to the benefits of carrying out a multiple resolution analysis [Eq. (1)]; we believe these benefits outweigh the perhaps erroneous fractal label. But let us first begin with the cutoffs of the limited range.

The existence of cutoffs is inherently associated with real physical objects experimentation. The lower cutoff is dictated typically either by the basic building block unit (atom, molecule, microcrystal, small aggregate etc.) of the system. The upper cutoff is, at most, of the order of the system size but usually much below it. It is bounded either by the me-

chanical strength, or by growth rates which drop sharply with time, or by the emergence of background effects such as non-isotropic fields, or by the depletion of resources. We recall here that many-orders scaling is found in temporal self-affine trails, but this is a completely different issue: the time axis can be extended at will.

Do very-limited range power laws represent fractals? Is it justified to term them as such (5)? It is our view that regardless of the question of fractality, the more basic question to be asked is: Is this presentation useful? The very existence of so many reports by competent researchers who are well aware of the problematics of declaring fractality for one order of magnitude experimental results, suggests that indeed, experimentalists seem to gain from the resolution analysis and from the fact that the result of such analysis is often a power law. The usefulness is in the following points:

- The power law condenses the description of a complex geometry.
- It allows one to correlate properties and performances of a system to its structure and to the dynamics of its formation, in a simple way.
- In many instances, the choice is either to use the limited range data, or to discard it all together and not to have even an approximate picture of the studied object. Opting for the former can be emphatically understood.
- Fractal geometry provided a proper language and symbolism which allowed the front-staging and the legitimization of studies of ill-defined geometries.

It is important to reiterate however that the ability to fit data to Eq.(1), does not imply fractality, and that the label "fractal" is not needed. So should one refer to such results in terms of a fractal object? If by "fractal" one refers to the original Mandelbrot teaching of many orders of magnitude, then the data we collected does not seem to support it in an unequivocal way. If by "fractal" one means an object that obeys Eq.(1) over a limited range, then the use of this label may be acceptable, not only because of its usefulness, but also because of the following additional reasons:

- Interestingly, the sense of self-similarity in irregular objects is comprehended visually even for a very limited range.

- In some cases, experimentally derived objects resemble simulative objects obtained from fractal models.

- The empirical D values for spatial objects fall in the fractal regime of $0 < D < 3$.

- And, it may be too late to make any changes in a terminology which, at this stage, seems to be deeply rooted in practice. A drift from an original meaning of a concept is common in science, representing adaptability of the original ideal definition to realistic restrictions which emerge when put to practice.

We arrive at our final question: Is then the Geometry of Nature, Fractal? Several key processes involving equilibrium critical phenomena (in magnets, liquids, percolations, phase transitions etc.) and some non-equilibrium growth models (aggregation), are backed by intrinsically scale free theories, and lead therefore to power law scaling behavior on all scales. However, the majority of the data which was interpreted in terms of fractality in the surveyed Phys. Rev. Journals, does not seem to be linked (at least in an obvious way) to existing models, and in fact, does not have theoretical backing. Most of the data represents results from non-equilibrium processes. The common situation is this: An experimentalist performs a resolution analysis and finds a limited range power-law, with a D value smaller than the embedding dimension. Without necessarily resorting to special underlying mechanistic argumentations, the experimentalist then often selects to label the object for which she/he finds this power law - "fractal" - this is the Fractal Geometry of Nature.

FIGURES

FIG. 1. See page 6 in Ref. [3]: A histogram of the number of decades of experimentally derived scaling exponents, which gave rise for declaring the studied system, fractal [4].

REFERENCES

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[2] R.V. Kalia *et. al*, Phys. Rev. Lett. **78**, 2144 (1997).

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[4] Our earlier version of this histogram [3] had two cases with a range of 3.7-3.8 decades. It turns out that we were too "liberal" in our interpretation: One case was a deterministically built exact Koch fractal (B. Sapoval *et. al*, Phys. Rev. E, **48**, 3333 (1993)) and the other was described by the authors as representing "almost no deviations... for the first 3 decades" (T. Holten *et. al*, ibid, **50**, 754 (1994)).

[5] For earlier critical analyses , see (a) L.P. Kadanoff, Phys. Today, Feb. 1986, p. 6; (b) O.R. Shenker, Stud. Hist. Phil. Sci. **25**, 697 (1994).